Computing with Bivariate Distributions

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- http://www.mynl.com/MALT/home.html

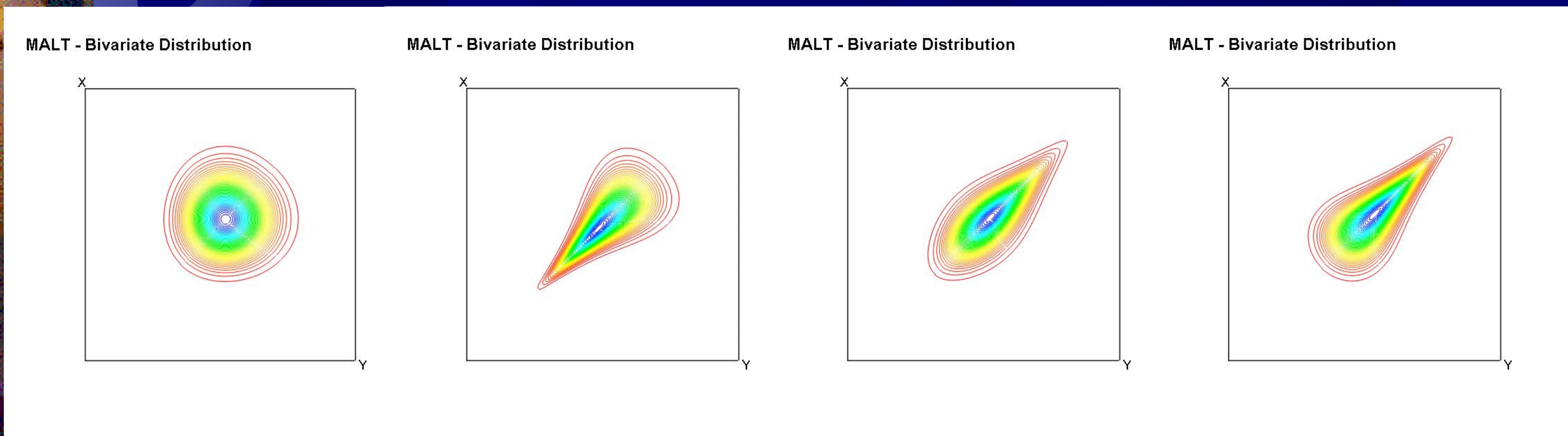
Copulas

- Venter Talk and Presentation
- If H(x, y) is a bivariate distribution then

$$H(x,y) = C(F(x), G(y))$$

where F, G are the marginals and C is a copula

Examples of Copulas

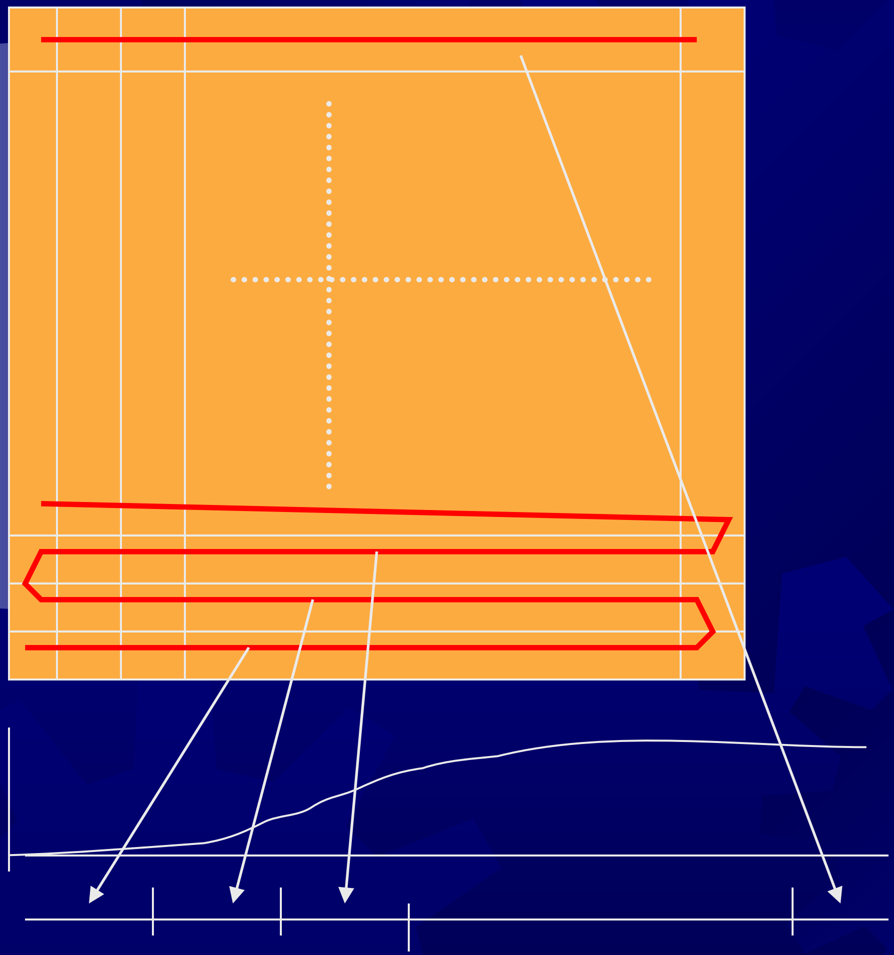


Independent Clayton Gumbel Venter HRT

Simulating From Copulas

- Univariate: $F^{-1}(u)$, u ~ Uniform
- Bivariate: doesn’t work
- Moments thought: tricky problem
- Venter: invert conditional and use two step method
- Normal Copula: Choleski decomposition

Simulating From Copulas



- Use space-filling curve to convert bivariate distribution into univariate distribution
- Sample off univariate distribution
- Convert back to bivariate distribution!

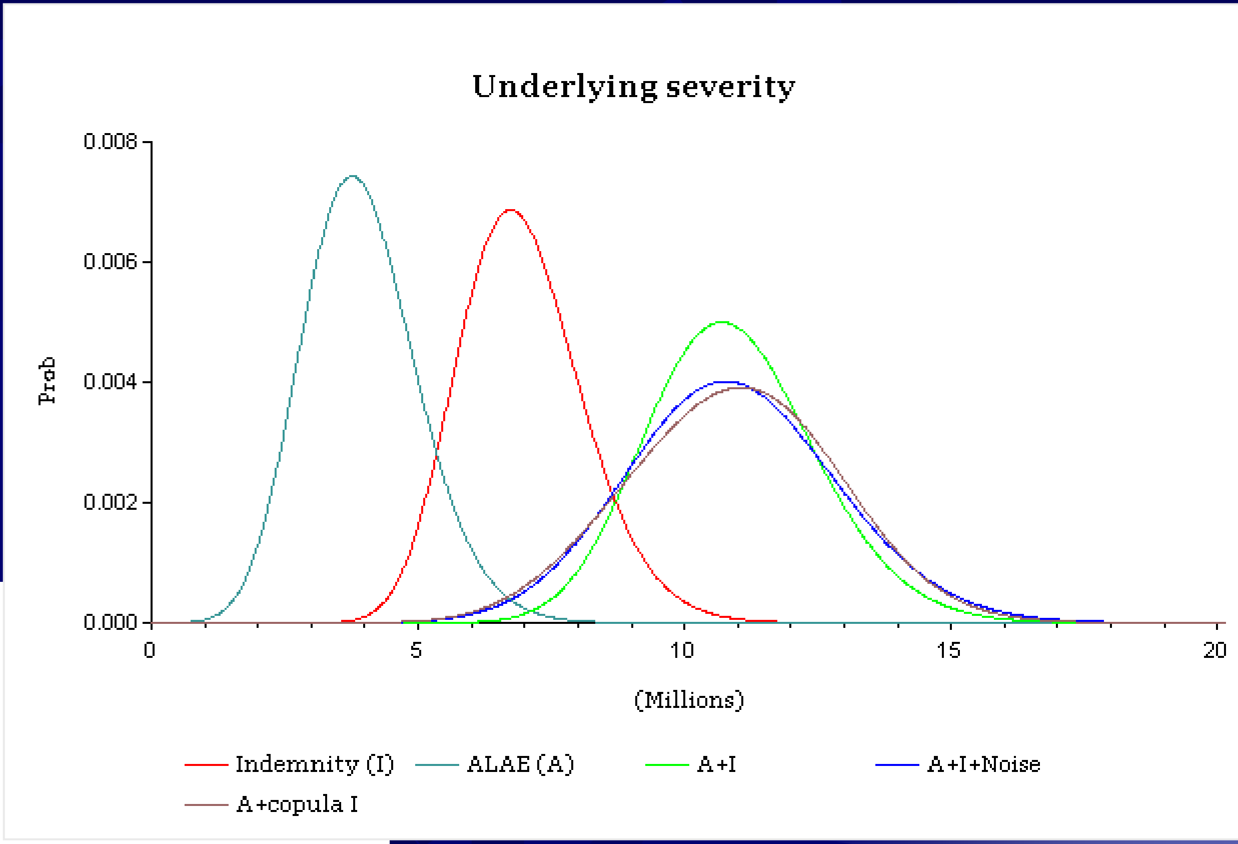
Convolution and Aggregates

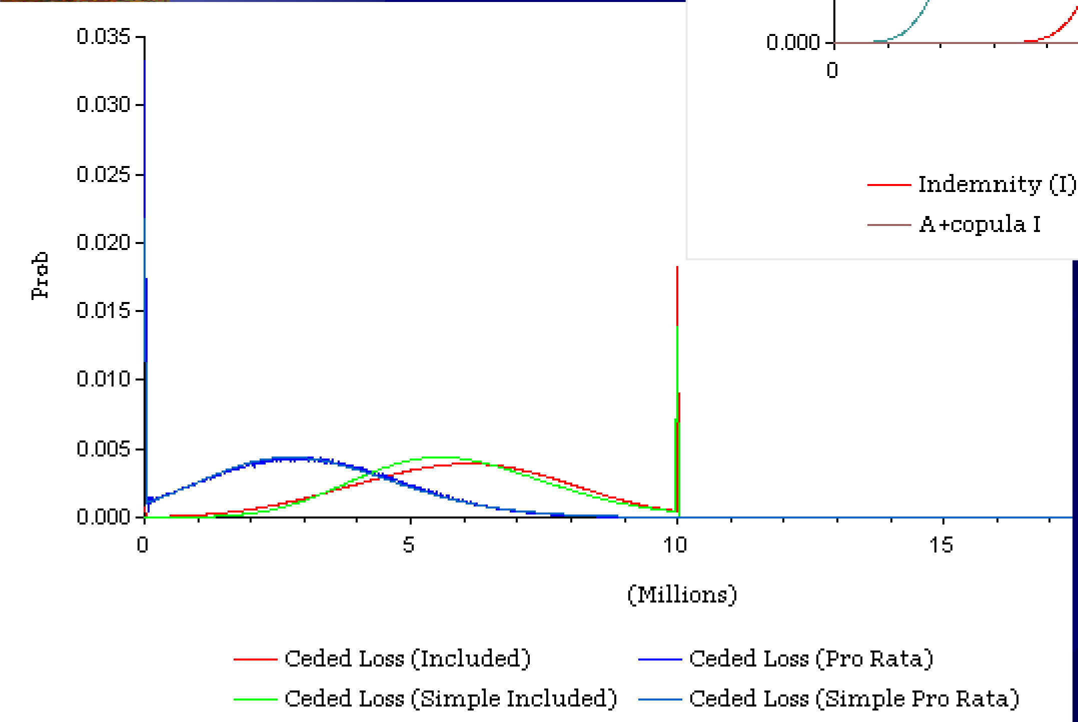
- X, Y random variables with MGFs $M_X(t)$ and $M_Y(t)$, then
- X+Y has MGF $M_{X+Y}(t)=M_X(t).M_Y(t)$
- If N is a frequency distribution and
  - $S = X_1 + \ldots + X_N$
- Then
  - $M_S(t) = M_N(\log(M_X(t))$
- Key Observation:
  - X, Y need not be 1-dimensional!!

Sum with Copula Dependence

- If (X,Y)~H(x,y) is a bivariate distribution
  - Marginals and copula specified
    - Cat losses in De, Md
    - Loss, ALAE
  - M = matrix “bucketed” sample from H
  - X+Y = IFFT(Diagonal(FFT2(M)))
    - FFT2 is two dimensional FFT
    - Not sensible, easier to sum diagonals
  - Can also use FFT methods to add white-noise to increase variance

Example: Loss & ALAE

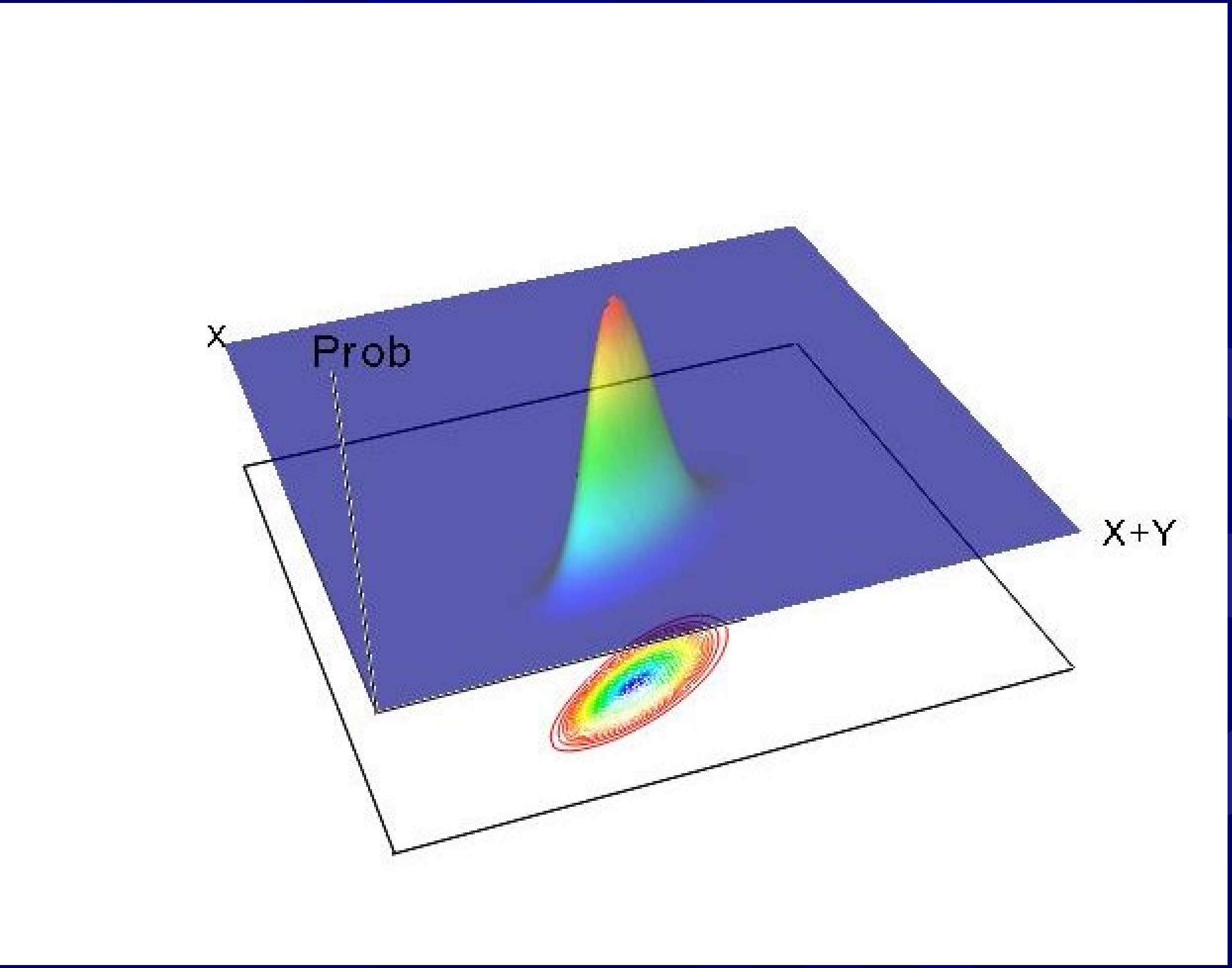




(Loss, Ultimate Loss)

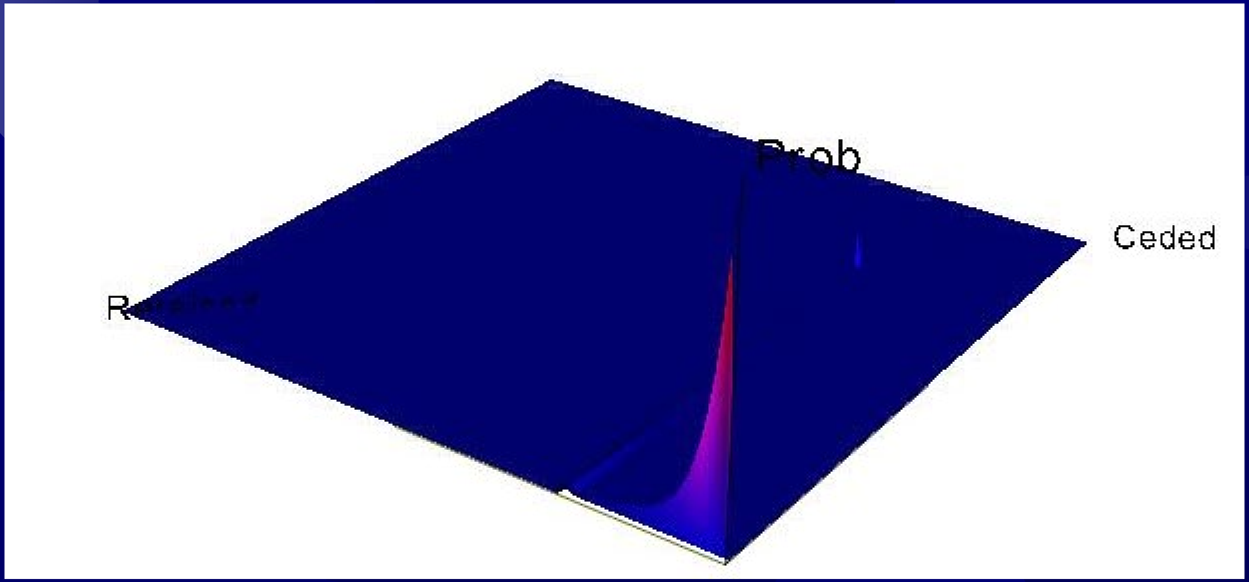
- General Problem: distribution of
  - (X,X+Y), where the X's are perfectly correlated
    - X(1,1) + Y(0,1)
    - X = incurred or paid loss
    - Y = bulk IBNR
- Use FFT techniques:
  - K = density of X along diagonal (matrix)
  - L = density of Y along Y axis (matrix)
  - IFFT2(FFT2(K).FFT2(L)) is required distribution

Loss, Ultimate



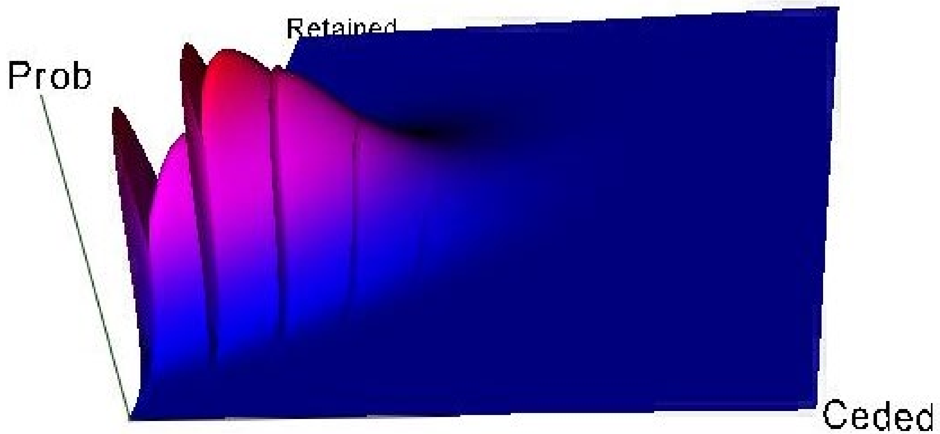
Net and Ceded

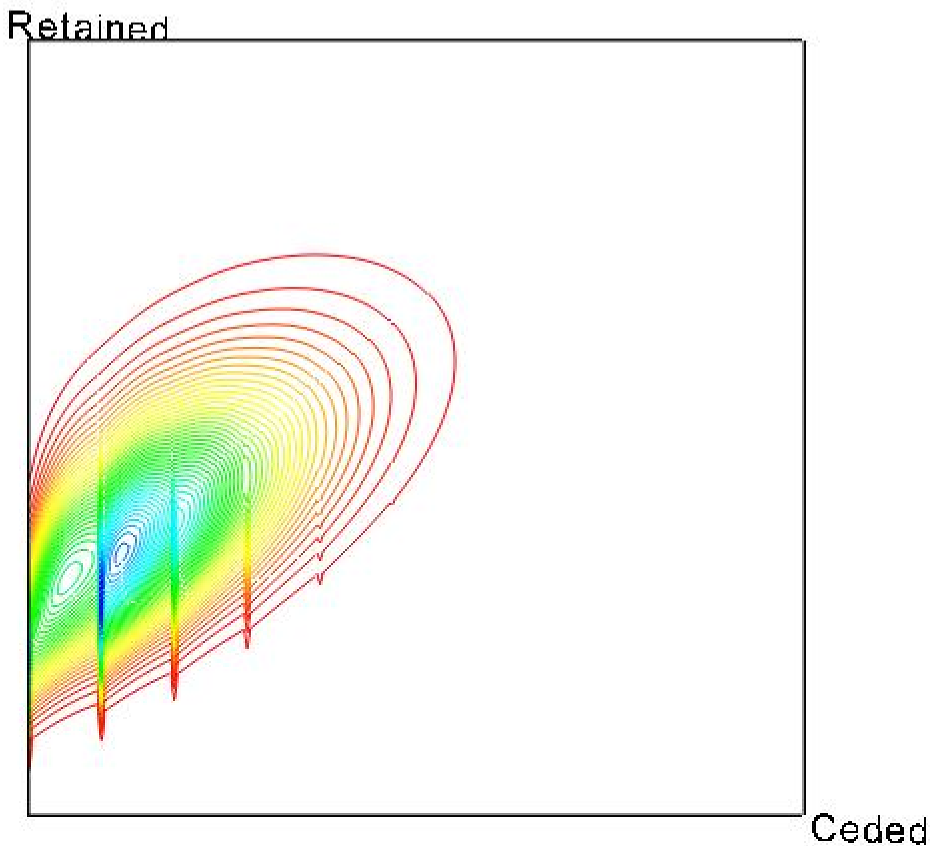
- Per occurrence cover: $1M policy limit, $50K deductible, 750K xs 250K ceded
- Per claim distribution:



Net and Ceded

- Apply claim count distribution using MGFs
  - 50 claims xs $50K expected
  - Neg. Binomial distribution, Var = 150





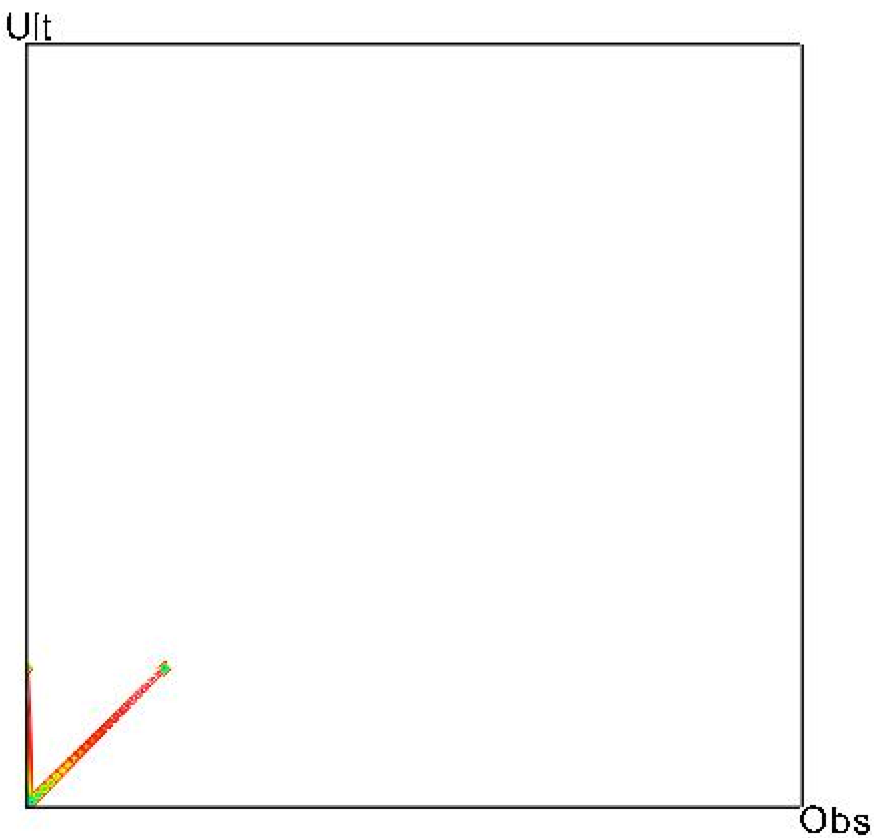
Paid Loss Development or (Loss, Ultimate) Redux

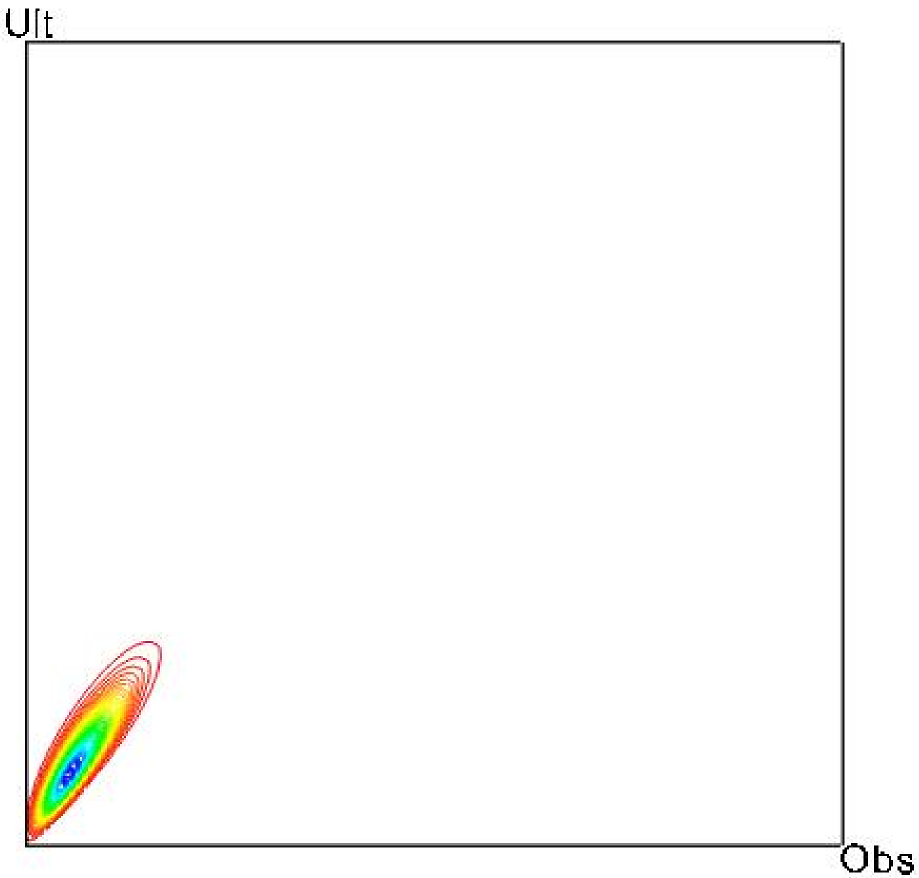
- At time n claim either paid or not paid

Per Claim Distribution

Scales are different!

Convolve with NB(50,300)





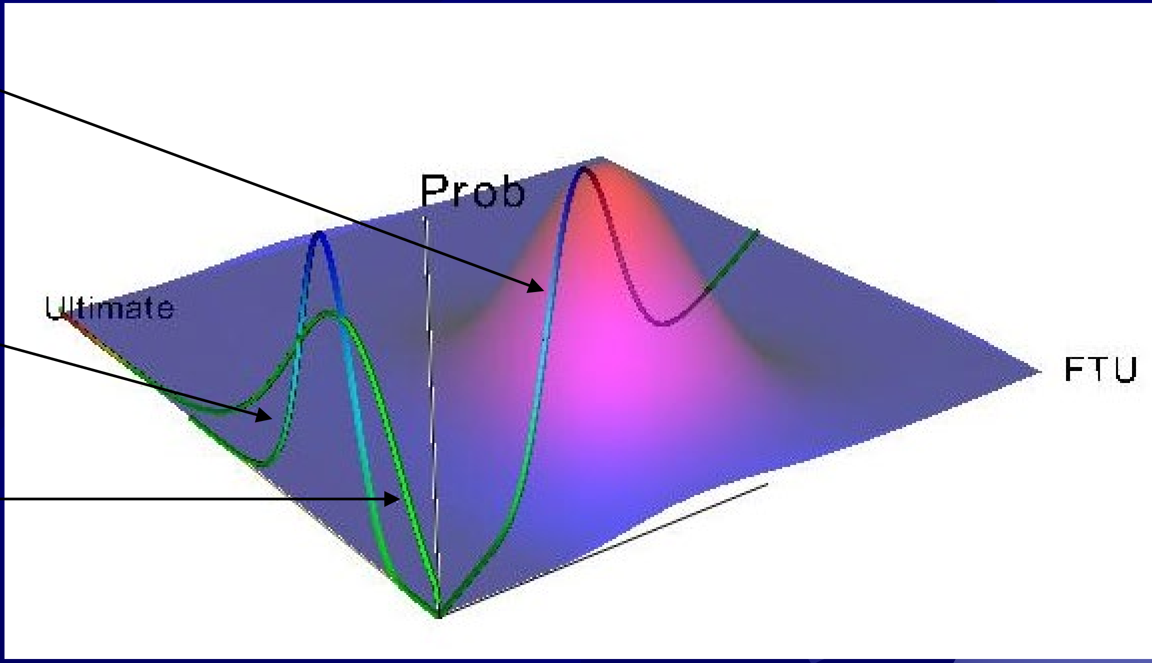
Paid Loss Bayesian Development

- Transform to Bivariate Dist of Ult vs FTU

Ult = FTU x Observed Loss

Posterior dist of ult losses given observed losses

Prior dist of ult losses



MALT - Ultimates for 1998